

## ПРИРОДНО-АНТРОПОГЕННЫЕ РЕЖИМЫ В ПРОСТЫХ МОДЕЛЯХ ГЛОБАЛЬНОГО МОРСКОГО РЫБОЛОВСТВА

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Исследование применяет метод системной динамики для развития моделей глобального морского рыболовства Всемирного банка в интересах достижения целей устойчивого развития ООН. Ключевыми переменными выступают глобальный морской рыбный запас, его естественный прирост, а также промысловое усилие и вылов рыбы. Выведены уравнения воспроизводства рыбных запасов в режимах максимально устойчивого вылова, перелова или недолова. Определены временные рамки до коллапса при характерных разновидностях перелова. Предложена новая положительная обратная связь в регулировании глобального морского рыбного запаса для асимптотического обеспечения максимально устойчивого вылова.

**Ключевые слова:** экосистемный подход к рыболовству, биоэкономическая модель, максимально устойчивые вылов и запасы, подвергающиеся перелову запасы, эксплуатируемые с недоловом запасы, регулирование рыболовства, обратная связь.

## NATURAL-ANTHROPOGENIC REGIMES IN SIMPLE MODELS OF THE GLOBAL MARINE FISHERIES

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The study applies the system dynamics method in upgrading the World Bank's global marine fisheries models in the interest of achieving the UN goals of sustainable development. The key variables are a global marine fish stock, its natural growth, as well as the fisheries' effort and catch. The equations for the reproduction of the fish stock in the regimes of maximum sustainable yield, overfishing or undercatch are derived. Determined are collapse time frames in result of typical overfishing varieties. A new positive feedback loop has been proposed in regulating global marine fish stock to ensure maximum sustainable yield asymptotically.

**Key words:** ecosystem approach, bioeconomic model, maximum sustainable yield, maximally sustainably fished stocks, overfished stocks, underfished stocks, fishery management, feedback loop.

## *Introduction*

The state of global marine fishery resources, based on FAO's monitoring of assessed marine fish stocks, has continued to decline [1, p. 6]. This requires substantial improvement of fishery management in the framework of ecosystem approach to renewable natural resources.

Substantial struggles are needed for achieving a target (14.4) set for marine fisheries among the UN Sustainable Development Goals [2, p. 24 (35)]: “By 2020, effectively regulate harvesting and end overfishing, illegal, unreported and unregulated fishing and destructive fishing practices and implement science-based management plans, in order to restore fish stocks in the shortest time feasible, at least to levels that can produce maximum sustainable yield as determined by their biological characteristics.”

The reader can explore [3, pp. 6–8, 60] as a valuable source for detailed explanation of the terminology and the established models in the field. Thereby the terms “depletion” and “depleted” describe respectively the process of decline in biomass and the state of a stock driven well beyond its level of maximum productivity (referred to as the maximum sustainable yield – MSY – level). The term “overfished” is a root term for “overexploitation”, “depletion”, “collapse” and may overlap with “rebuilding” and “recovery”.

Depletion is graded depending on severity. Collapses occur because of excessive fishing, natural climate-induced calamities or both. Acute depletion may lead to collapse and even to extinction as a possible extreme outcome if rebuilding is not undertaken in time.

The terms “recovery” and “rebuilding” have been used interchangeably in the stock assessment literature. In the aggregated research approach of the present paper, the rebuilt or restored state is achieved whenever an overfished stock has been re-established to the rebuilding target level (particularly,  $X_c$  in Table 3 below).

In disaggregated studies, the term “rebuilding” is more demanding: it means a more comprehensive re-establishment of particular depleted stocks, including age structures, evolutionary mechanisms, and population traits. Rebuilding is assumed to require much more time to be achieved than time needed to recover or restore overfished stocks.

According to the control theory, open-loop control is completely determined at the initial instant  $t_0$ ; here, the integration of the equation (or equations) of motion for fixed initial conditions defines the phase trajectory  $x(t)$  of the states of the system [4]. Closed-loop control (with feedback) assumes the definition of control as a function of phase coordinates and time (*ibid.*). These concepts have wide theoretical and applied significance for bioeconomic theory and practice, in particular.

Science-based management plans are at the core of an adaptive management process that includes regulative feedback loops at different time scales based on past and present observations and experiences. This requires equilibrium and, especially, non-equilibrium system dynamics models of global marine fisheries. Such models have to propose policies (harvesting control rules) that can bring global marine fisheries from worrying disequilibrium closer to a state that supports MSY.

A World Bank's complete bioeconomic model (WB model) allows assessing the sustainability of the global marine fishery [5]. It contains endogenous variables involved in feedback loops: biomass, gross and net biomass growth, harvest. Several economic variables are not engaged in feedback loops: fishing effort, fish price, costs, revenues and net benefits. This model is used, in particular, for comparing how fast fisheries' recovery can be expected depending on the fishery's governance structure supported by interwoven feedback loops for endogenous variables.

This brief paper analyses a reduced WB model that abstracts from fish price, costs, revenues and net benefits. This model, supported by real data and by application experiences, has a potential for significant modification. The serious disadvantage of this model (as well as of the complete WB model) is exogenous fishing effort to be healed in an upgraded model. The notion of MSY is supplemented by notion of fitting sustainable effort (FSE) in this paper.

W-1, W-2, W-3 and finally W-4 are acronyms for the models considered. A higher number corresponds to a higher degree of sophistication measured by a quantity of feedback loops involved. Therefore, each subsequent model generalizes the preceding one. The reduced WB model is denoted as W-3 hereby.

The author uses the concept of aggravation modes (regimes), investigated in different contexts, in particular, in [5–9]. Especially dangerous are those arising from dominance of the positive feedback connecting biomass and the rate of its net change when the biomass diminishes faster and faster. On the other hand, dominance of such positive feedback loop can foster recovery of the depleted stock up to the point when a maximal positive growth rate of biomass is reached. Afterwards the growth rate of biomass asymptotically declines to zero when the biomass approaches stock that can open-endedly support MSY as in W-3 and W-4 below.

### *Natural net growth of biomass in W-1*

Fish hatch (give birth), grow to maturity, lay eggs and die. Fish death rate is the number of fish per year that die from causes other than fish harvesting. Factors of fish population simple growth are reflected by the Pella-Tomlinson net biomass growth function as a specific non-linear autonomous differential equation [5, 10 and 11]:

$$\dot{x} = \phi(x) = \alpha x - \beta x^\gamma. \quad (1)$$

The initial assumptions for (1) are as follows: the rate of reproduction of the population is proportional to its current level; the second term of the equation reflects intraspecific competition for resources, which limits the growth of the population, or, in plain words, the death rate increases as crowding increases.

If  $\gamma = 2$ , the Pella-Tomlinson function becomes the well-known logistic function [12], and as  $\gamma$  approaches unity, it converges to the Fox biomass growth function [13].

Tables 1 and 2 reflect variables and parameters of W-1 and subsequent models.

Table 1

## The variables of the biomass models

Variable	Notation	Measurement unit
Catch	$y, c$	fish mln tons / year
Fish stock (biomass)	$x$	fish mln tons
Carrying capacity $X$	$(\alpha/\beta)^{1/(\gamma-1)}$	fish mln tons
Birth rate	$\alpha x$	fish mln tons / year
Death rate	$\beta x^\gamma$	fish mln tons / year
Net change of fish stock	$\dot{x}$	fish mln tons / year
The growth rate of fish stock	$\hat{x}$	1/year
The growth rate of catch	$\hat{y}$	1/year

Table 2

## The parameters and base-year quantities

Characterization	Values	How obtained
Biological coefficients		
Intrinsic growth rate $\alpha$	1.644	Calculated
$\beta$ (component of death rate)	0.45	Calculated
Pella-Tomlinson exponent $\gamma$	1.188	Estimated
Bioeconomic coefficients		
Catchability $q$	1.76	Calculated
Schooling parameter $b$	0.71	Estimated
Base-year (2012) quantities		
Fishing effort $e(2012)$	1	Normalized
Biomass $x(2012)$ , fish mln tons	214.9	Calculated
Landed quantity $y(2012)$ , fish mln tons / year	79.69	Estimated

The derivative of the natural net change is defined as

$$\phi'_x = \alpha - \beta\gamma x^{\gamma-1}. \quad (2)$$

The stationary states are found from the condition that the right-hand side of (1) is equal to zero. They differ qualitatively and quantitatively.

On the one hand, carrying capacity  $x_1 = X$  is asymptotically stable node, since  $\phi'_x(x_1) = \phi'_x(X) = \alpha(1 - \gamma) < 0$  for  $\gamma > 1$ , on the other hand,  $x_2 = 0$  is unstable node, as  $\phi'_x(x_2) = \alpha > 0$ .

The population growth is S-shaped. The biomass tends to  $X$  that can sustain most of random external shocks except huge calamities. W-1 is structurally stable.

### ***Exogenous catch in W-2***

W-2 additionally assumes a harvesting control rule implying that human fishing activities reduce the increase in the fish population by catch amount  $y = c = \text{const} \geq 0$ :

$$\dot{x} = f(x) = \phi(x) - c. \quad (3)$$

Next equation defines the rate of growth of the stock

$$\hat{x} = \alpha - \beta x^{\gamma-1} - c / x, \quad (4)$$

where the hyperbolic element is potent of biomass extinction through aggravation mode for some  $c > 0$ . A birth of the aggravation mode results from the transition from dominant negative feedback  $x \xrightarrow{-} \hat{x} \rightarrow \dot{x}$  to dominant positive feedback  $x \rightarrow \hat{x} \rightarrow \dot{x}$  at a tipping point, when the sign of  $\partial \hat{x} / \partial x = -\beta(\gamma-1)x^{\gamma-2} + c / x^2 < 0$  turns into its opposite. Quite dramatically for  $x \rightarrow 0$   $\hat{x} \rightarrow -\infty$  and  $\partial \hat{x} / \partial x \rightarrow +\infty$ . Notice that  $x = 0$  reflecting extinction is not literally mathematical equilibrium.

W-2 can possess two, one or no stationary states depending on the parameters values. Consider one distinct stationary state at first.

Lemma 1. The line  $y = c > 0$  is tangential to the curve of  $\phi(x)$  at  $X_c$  with  $\phi'_x = 0$  and  $\phi''_x < 0$  – here net increment  $\phi(x)$  is globally maximal. Besides  $\phi'_x > 0$  for  $x < X_c$  and  $\phi'_x < 0$  for  $x > X_c$ . Maximal catch is  $Y_c = \phi(X_c)$  (Table 3).

*Table 3*

Equilibriums for contrast natural-anthropogenic regimes in the models

Variable	W-2 (saddle), W-3 (stable node) and W-4 (stable node)	W-3 (saddle) and W-4 (unstable node)
Stock $x$	$X_c = \left( \frac{\alpha}{\beta\gamma} \right)^{\frac{1}{\gamma-1}} = 391.98$ Maximally sustainably fished in W-3, W-4	$x_c = \left[ \frac{\alpha(1-b)}{\beta(\gamma-b)} \right]^{\frac{1}{\gamma-1}} = 68.68$ Unsustainable
Catch $y$	$Y_c = \alpha \frac{\gamma-1}{\gamma} X_c = 102$ MSY in W-3 and W-4	$y_c = \frac{\beta(\gamma-1)}{1-b} x_c^\gamma = 44.42$ Unsustainable
Fishing effort $e$	$E_c = \frac{\alpha \gamma-1}{q \gamma} X_c^{1-b} = 0.8354$ FSE in W-3 and W-4	$e_c = \frac{\beta(\gamma-1)}{q(1-b)} x_c^{\gamma-b} = 1.2528$ Unsustainable

Proposition 1. The stationary state for maximal catch  $Y_c$  is  $X_c$ .

Proof. Apply Lemma 1 and notice that  $f(X_c) = 0$ .

Corollary 1. No stationary state exists if  $y = \text{const} > Y_c$ . Available biomass  $x$  decreases from  $x_0$  to its elimination through aggravation mode.

For example, for  $x_0 = X = 980$  and  $c = 110 > Y_c = 102 > y_0$  it takes 45 years until extinction through aggravation mode.

Pay attention to two distinct stationary states.

Proposition 2. Let  $0 < c < Y_c$ . The stationary states are defined as  $0 < x_2 < X_c < x_1$ . Proof. Thanks to the properties of  $\phi(x)$  the line  $0 \leq y = c < Y_c$  intersects the curve of function  $\phi(x)$  twice in stationary states  $0 < x_2 < x_1$ .

Corollary 2. Lower stationary state  $x_2$  is unstable node, since  $f'_x(x_2) > 0$ , while higher stationary state  $x_1$  is stable node, since  $f'_x(x_1) < 0$ .

Biomass falls from  $x_0 > 0$  to bottommost through aggravation mode whenever  $x_0 < x_2$ . Biomass decreases from  $x_0$  to  $x_1$  if  $x_0 > x_1$ , it increases to  $x_1$  if  $x_2 < x_0 < x_1$ .

Let  $y = y_0 < Y_c$ . There are two equilibriums: unstable node  $x_2 = 175.33 < x_0 < x_1 = 648.22$ . The latter is stable node.

Biomass available in 2012 was not destined for collapse under these assumptions. Fixing global catch  $y_0$  would facilitate the biomass growth up by factor of 3 asymptotically. On the other hand, extinction, for example, could result from  $0 < x_0 = 174 < x_2 = 175.33$  after 18.9 year.

It is easy to see that both stationary states merge into one  $X_c$  if  $y = Y_c$ . There is a catastrophic change in the system's regime in response to a smooth change of this control parameter as in the Schaefer – Arnold model [12, 14 and 15].

Proposition 3. For  $y = Y_c = \alpha(\gamma - 1)X_c / \gamma$ , a saddle-node bifurcation takes place. This saddle-node is unstable for  $x < X_c$  and is stable for  $x > X_c$ .

Proof. The necessary and sufficient conditions for the saddle-node bifurcation are fulfilled [16, pp. 84–86]: the fusion of the nodes with the conversion into the saddle is confirmed by the inversion of the derivative at the critical point to zero  $f'_x(X_c, Y_c) = 0$  in the absence of degeneracy in it,  $f''_x(X_c, Y_c) \neq 0$ , and it is additionally supported by transversality condition  $f'_y(X_c, Y_c) = -1 \neq 0$  satisfied.

For the lower (unstable) branch of solutions  $x < X_c$  derivative  $f'_x(x, Y_c) > 0$ , whereas for the upper (stable) branch of solutions  $x > X_c$  derivative  $f'_x(x, Y_c) < 0$ . In other words,  $X_c$  is an attractor for  $x > X_c$  and a repeller for  $x < X_c$ .

Consider saddle equilibrium and saddle-node bifurcation for the given parameters magnitudes:  $y = Y_c > y_0$ ,  $X_c = 391.98 > x_0$ . There are stable branch for  $x > X_c$  and unstable branch for  $0 \leq x < X_c$ . The constant catch that is only 28 per cent higher than the observed one in 2012 would inevitably entirely deplete the global marine fish biomass within about 6.5 years.

### ***Catch dependence on biomass in reduced WB model W-3***

Now a harvesting function, or a harvesting control rule, is defined as

$$y = E(x) = qex^b \quad (5)$$

with fishing effort  $e = \text{const}$ . If  $b = 0$  then abnormal W-3 is identical to W-2.

W-3 differs from W-2 substantially for  $0 < b < 1$  as it includes new negative feedback loop B2 (Table 4). This structural change is stabilizing for global marine fisheries, as the reader will soon see.

*Table 4*

Three feedback loops in W-3

Loops descendant from W-1 and W-2	New loop
R1 of length 1 Stock $x \rightarrow$ Birth rate	B2 of length 1 Stock $x \rightarrow$ Catch $y$
B1 of length 1 Stock $x \rightarrow$ Death rate	

There is non-linear dependence of the rate of growth of the stock on itself

$$\hat{x} = \alpha - \beta x^{\gamma-1} - qex^{b-1}, \quad (6)$$

where hyperbolic element  $-qex^{b-1}$  is potent of aggravation mode leading to extinction. Indeed,  $\hat{x} \rightarrow -\infty$  for  $x \rightarrow 0$  as  $b < 1$  and  $\partial \hat{x} / \partial x = -\beta(\gamma - 1)x^{\gamma-2} + (1 - b)qex^{b-2} \rightarrow +\infty$  for  $x \rightarrow 0$ .

W-3 can possess one, two, or three stationary states depending on the parameters values. Of course, only for  $x_s = 0$  and  $x_s = X$  it is true that  $y_s = 0$  and  $e_s = 0$ . Otherwise, for given stationary state  $x_s$  there can be one or two  $y_s$  and co-responding one or two  $e_s$  for each  $y_s$ . This disjointedness prohibits direct extension of above Propositions from W-2 to W-3.

Notice that in W-3, contrary to W-2, for  $e_s > 0$ ,  $x_d = 0$  is broadly asymptotically stable (BAS) stationary state as  $f(0) = 0$  and  $f'_x(x) \rightarrow -\infty$  for  $x \rightarrow 0$ . Similarly,  $x_s = X$  is BAS stationary state as  $f(X) = 0$  requires  $e_s = 0$  and  $f'_x(X) = \alpha(1 - \gamma) < 0$ .

Stationary catch effort  $e_s \geq 0$  can be uniquely defined for known stationary biomass  $x_s \geq 0$  by (5). However, the reverse is not true: for the same stationary catch effort  $e_s \geq 0$  there can be one, two or three stationary biomass magnitudes  $x_s \geq 0$ .

Lemma 2. The line  $0 < e_s = e_c$  is tangential to the curve of function  $e_s(x_s)$  at its global maximum in  $x_c$  (Table 3).

Proof. Function  $e_s(x_s)$  achieves in  $x_c$  its global maximum  $e_c(x_c) = e_c$ . Indeed,  $e'_c = 0$  when  $\alpha(1-b)/[\beta(\gamma-b)] = x_c^{\gamma-1}$ , besides that  $e''_c < 0$  and  $e'_s > 0$  for  $x_s < x_c$ ,  $e'_s < 0$  for  $x_s > x_c$ .

Corollary 3. No strictly positive equilibrium exists if  $e > e_c$  (Table 3) since line  $e_s = \text{const} > e_c$  goes wholly strictly above the curve of function  $e_s(x_s)$ . The only equilibrium is globally asymptotically stable node  $x_d = 0$ . The available biomass declines from  $x_0$  to zero. A biomass collapse through aggravation mode is the consequence of a persistent over-stretched effort.

For example, for  $x_0 = 214.9$ ,  $e_0 = 1.26 > e_c = 1.253$  it takes about 200 years for extinction of the fish despite parallel decline in catch  $y$  from  $y_0 = 100.4$  to zero.

Proposition 4. For  $0 < e_s < e_c$  there are three stationary states: BAS node  $x_d = 0$ , unstable node  $0 < x_2 < x_c$  and stable node  $x_c < x_1 < X$ .

Proof. The line  $0 < e_s = \text{const} < e_c$  intersects the curve of function  $e_s(x_s)$  in two points corresponding to  $x_1$  with  $f'_x(x_1) < 0$  and to lower  $x_2$  with  $f'_x(x_2) > 0$ .

For example, take  $e_s = E_c < e_c$  (Table 3). There are three equilibriums:  $x_d = 0$  is BAS node,  $x_2 = 2.71$  is unstable node,  $x_2 < x_c < x_1 = X_c$ ,  $x_1$  is stable node. We see  $X_c$ ,  $Y_c$  and  $E_c$  are also uniquely defined. If the effort in 2012 and in subsequent years was fixed at  $E_c$ , the biomass would grow up by factor of 1.82 asymptotically from  $x_0$ .

Quantitatively the same  $X_c$  that is saddle equilibrium in W-2 becomes broadly stable node  $x_1$  instead thanks to negative FB loop B2 (Table 4).

Proposition 5. For  $e = e_c$ , a saddle-node bifurcation takes place. There are two equilibriums for  $e_c$  in W-3: stable node  $x_d = 0$  and saddle  $x = x_c$  (Table 3).

Proof. Indeed  $f(0) = 0$  and  $f'_x(x) < 0$  for  $x$  close to  $x_d = 0$ .

The necessary and sufficient conditions for the saddle-node bifurcation are fulfilled [16, pp. 84–86]: the fusion of the nodes  $x_1$  and  $x_2$  with the conversion into the saddle  $x_c$  is confirmed by the inversion of the derivative at the critical point to zero  $f'_x(x_c, e_c) = 0$  in the absence of degeneracy in it,  $f''_x(x_c, e_c) = \alpha(1-b)(1-\gamma)/x_c < 0$  and with satisfied transversality condition  $0 > f'_e(x_c, e_c) = -qx_c^b \neq 0$ .

This saddle-node is unstable for  $x < x_c$  and is stable for  $x > x_c$ . For the lower (unstable) branch of solutions  $x < x_c$  the derivative  $f'_x(x, e_c) > 0$ , whereas for the upper (stable) branch of solutions  $x > x_c$  the derivative  $f'_x(x, e_c) < 0$ . In other words,  $x_c$  is an attractor for  $x > x_c$  and a repeller for  $x < x_c$ .

Compare properties of saddle  $x_c$  in W-3 to those of saddle  $X_c$  in W-2 (Table 3).

Lemma 3. If  $b < 1$ ,  $\gamma > 1$  then  $\frac{1-b}{\gamma-b} < \frac{1}{\gamma}$ .

Corollary 4. The following subordination is true:  $x_c < X_c$  and  $y_c < Y_c$ .

Corollary 5. The equality  $X_c = x_c$  is not possible in W-3.



Proof. Indeed,  $\left(\frac{\alpha}{\beta\gamma}\right)^{\frac{1}{\gamma-1}} = \left[\frac{\alpha(1-b)}{\beta(\gamma-b)}\right]^{\frac{1}{\gamma-1}}$  if  $b = 0$  (as in W-2) or  $b > 0$  and

$\gamma = 1$  that is excluded in W-3.

Corollary 6. If  $x = x_d = 0$ , then  $y_s = 0, e_s = 0$ ; if  $0 < x_s \leq X$ , then  $y_s = \alpha x_s - \beta x_s^\gamma, e_s = y_s / q x_s^b \geq 0$ . In particular,  $y_s(X) = 0, e_s(X) = 0$ .

Thus, we have considered peculiarities of the three cases of stationary biomass magnitudes for given stationary catch effort: one equilibrium case, two equilibriums case, and three equilibriums case in W-3. New feedback loop B2 enhances sustainability of maximal catch  $Y_c$  in particular, through transformation of corresponding saddle  $X_c$  in W-2 into stable node  $x_1 = X_c$  higher than new saddle  $x_c$  in W-3.

### *Effort dependence on target biomass in W-4*

W-1 and W-3 are from [5], W-2 and W-4 as their modifications are developed in this paper. Notice that the base-year fishing effort  $e$  exceeds FSE, biomass  $x$  is over-fished; catch  $y$  is lower than MSY (Table 3).

Let recovery of biomass capable of robust maximal sustainable yield  $Y_c$  be the goal of stabilization policy. Robustness means that even if the global fishing effort  $e_0$  equals  $e_c$  the biomass will not plunge to  $x_c$  from  $x(2012)$  as is inevitable in W-3 but will climb to  $X_c$  in modified model W-4. Parameter fishing effort  $e$  is transformed into variable  $e$ . This means that W-4 generalises W-3 as its special form.

The causal loop structure of W-4 augments causal loop structure of W-3 with new positive feedback loop that includes new variable  $e$  (Table 5).

*Table 5*

Four feedback loops in W-4

Loops descendant from W-3	New loop – positive
R1 of length 1 Stock $x \rightarrow$ Birth rate	R2 of length 2 Stock $x \xrightarrow{-}$ Effort $e \rightarrow$ Catch $y$
B1 of length 1 Stock $x \rightarrow$ Death rate	
B2 of length 1 Stock $x \rightarrow$ Catch $y$	

This structural upgrading, not suggested in [5], enables simultaneously needed destabilization of former W-3 saddle  $x_c$  by turning it into W-4 unstable node and – at the same time – by keeping W-3 stable node  $X_c$  as stable node in W-4 too.

Next harvesting control rule is a reasonable substitute for (5) in W-3:

$$y = F(x) = qex^b = q\varepsilon x^{h+b}, \quad (7)$$

where variable effort  $e$ , new constants  $\varepsilon$  and  $h$  are determined as

$$e = e_0(x/x_0)^h = \varepsilon x^h, \quad (8)$$

$$\varepsilon = e_0/x_0^h > 0, \quad (9)$$

$$h = \ln(E_c/e_0)/\ln(X_c/x_0) < 0. \quad (10)$$

The growth rates of biomass, effort and catch are defined next

$$\hat{x} = \alpha - \beta x^{\gamma-1} - q\varepsilon x^{h+b-1}, \quad (11)$$

$$\hat{e} = h\hat{x}, \quad (12)$$

$$\hat{y} = (h+b)\hat{x}. \quad (13)$$

Hyperbolic element  $-q\varepsilon x^{h+b-1}$  in (11) is potent of aggravation mode. Indeed, as  $h+b < 1$  consequently  $\hat{x} \rightarrow -\infty$  for  $x \rightarrow 0$  and  $\partial\hat{x}/\partial x \rightarrow +\infty$  for  $x \rightarrow 0$ .

Lemma 4. Let  $e_0 = e_c$ ,  $y_0 = y_c$  and  $x_0 = x_c$ . Then in agreement with (10)  $h_c = \ln(E_c/e_c)/\ln(X_c/x_c) < 0$ .

Proposition 6. For these parameters, there are three equilibriums in W-4: BAS node  $x_d = 0$ , unstable node  $x_2 = x_c$  and stable node  $x_1 = X_c$ . Corresponding effort and catch are  $e = 0, y = 0$  for  $x_d$ ,  $e_c$  and  $y_c$  – for  $x_2$ , finally  $E_c$  and  $Y_c$  – for  $x_1$ .

Proof. Check stationarity and stability (instability) of these three points:  $f(0) = 0$ ,  $f'_x(x) \rightarrow -\infty$  for  $x \rightarrow 0$  therefore  $x_d$  is BAS node as in W-3;  $f(x_c) = 0$  by definition of  $x_c$  for  $e = e_c$ ,  $f'_x(x_c, e_c) > 0$  therefore  $x_c$  is unstable node contrary to W-3;  $f(X_c) = 0$  and  $f'_x(X_c, E_c) < 0$  due to Lemma 4 therefore  $X_c$  is stable node as in W-3.

For the given data,  $\varepsilon_c = 3.351$  and  $h_c = -0.2326$ . If initially  $e(0) = e_c = 1.253$ ,  $x(0) = 68 < x_c = x_2$ , the biomass plunges to extinction accompanied by relentless growth of effort  $e$ ; for  $x(0) = 70 > x_2$  the biomass climbs to stable node  $x_1 = X_c$  while effort  $e$  diminishes to  $E_c$  (Tables 3 and 6).

If non-equilibrium biomass, catch and effort observed in 2012 are used instead then  $\varepsilon = 4.989$ ,  $h = -0.2993$  and  $x_2 = 106.4$ . In the satisfying solution, biomass  $x$  and catch  $y$  will climb for 40 years asymptotically from 54.8 to 98.6 per cent of  $X_c$  and

from 78.1 to 99.4 per cent of  $Y_c$  whereas effort  $e$  will asymptotically decline from 119.7 to 100.4 per cent of  $E_c$ . This targeted transition is moderate in pace.

Table 6

Specific equilibriums in W-2, W-3 and W-4

Equilibrium	W-2		W-3		W-4 (for $h_c$ )
Stock $x_d$ (BAS node)	(not present)	(not present)	0	0	0
Stock $x_2$ (unstable node)	175.33 Overfished	$x(2012) = 214.9$ Overfished	7.26 Overfished	13.61 Overfished	$x_c = 68.68$ Overfished
Stock $x_1$ (stable node)	648.22 Underfished	594.31 Underfished	283.14 Overfished	$x(2012) = 214.9$ Overfished	$X_c = 391.98$ Maximally sustainably fished
Catch $y$ for $x_1$	$y(2012) = 79.69$ (const) < MSY	87.7 (const) < MSY	96.93 < MSY	$87.7 < MSY$	$Y_c = 102$ MSY
Effort $e$ for $x_1$	(implicit $y_0/(qx_1^b)$ ) $0.457 < FSE$	(implicit $y_0/(qx_1^b)$ ) $0.535 < FSE$	$e(2012) = 1$ (const) > FSE	$1.1$ (const) > FSE	$E_c = 0.8354$ FSE

**Conclusion**

This study augments W-3 by a revitalising positive feedback loop in W-4. This loop implies endogenously reduced fishing effort to rebuild the overfished fish stock.

The harvesting control rules are developed, firstly, for avoiding extinction of the global marine fish through typical aggravation modes in W-2 and W-3 and, secondly, for attaining maximal sustainable yield (MSY) and fitting sustainable effort (FSE) asymptotically in W-4.

The disequilibrium initial state of the 2012 year has had margins of safety (Table 6) favorable for recovery of the overfished stock under the proposed harvesting control rules. Catching more fish with the same or less fishing effort could happen due to the organized increases in the biomass in W-2, W-3 and W-4.

Control solutions with jump discontinuity in fishing effort for a most rapid path based on Pontryagin’s maximum principle [4, 5, 17 and 18] will be substituted by smooth proportional and derivative control in original two-dimensional predator – prey models in line with [15]. Transition time from  $y(2012)$  to vicinity (99 per cent) of MSY could be reduced under urgency from about forty years in the satisfying solution in W-4 to roughly ten years through parametric policy optimization in the predator – prey models. Social and political costs associated with altering the way fisheries are operated will be taken into fuller account than in the neo-liberal policy proposals grounded on mainstream (“neoclassical”) models.

A later work will also address system dynamics aspects of fish price, costs, revenues and net benefits touched in [5] still without mentioning transnational corporations as key actors in marine ecosystems. Choking this omission requires deepening the simple functional relationships between fish replenishment and human fishing activities.

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